BLACK HOLES AND LARGE-c BCFTS

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Based in part on arXiv: 1810.10601 (w/ Cooper, Rozali, Swingle, Van Raamsdonk and Waddell)

Introduction

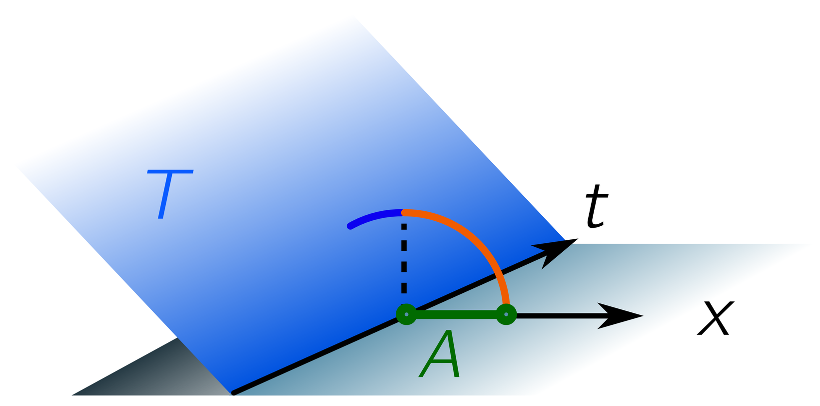
Consider a 2d CFT in a pure state. **Quenching** is a non-equilibrium process where we **suddenly change** the Hamiltonian and let the system thermalise. If the state is a conformally invariant **boundary state**, the quench can be analysed using **boundary CFT (BCFT)**. In particular, we can calculate **entanglement entropies (EE)**.

According to **AdS/BCFT**, the dual geometry is a **black hole**. We calculate **holographic EE**, and **find agreement** with the BCFT result in some cases. This **provides evidence** for AdS/BCFT, and **constrains holographic BCFTs**, simultaneously. Our analysis also hints at how **behind-the-horizon physics** is encoded in the BCFT.

AdS/BCFT and black holes

A 2d BCFT is a CFT on a half space $x > 0$ with conformally invariant boundary conditions. The EE of an interval $A = [0, L]$ is half the CFT result on $[-L, L]$, plus a novel **boundary entropy** term [1]:

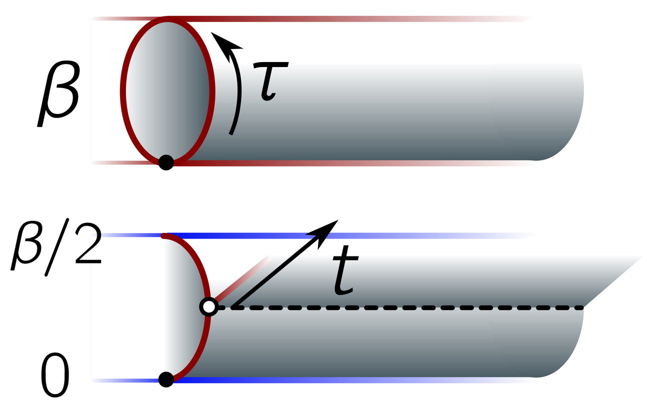
$$S_A = -\mathrm{Tr}[\rho_A \log \rho_A] = \frac{c}{6}\log\frac{2L}{\epsilon} + \log g_B.$$

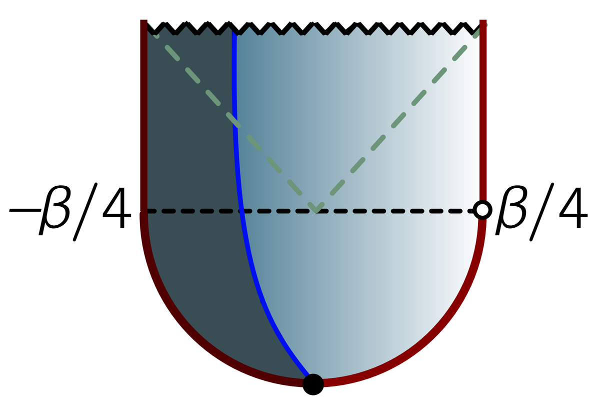


The AdS/BCFT dictionary [2] proposes that a holographic BCFT is dual to AdS cut off by a **brane at fixed tension** T, where

$$\log g_B = \frac{c}{12}\log\left[\frac{1+\ell_{\mathrm{AdS}}T}{1-\ell_{\mathrm{AdS}}T}\right].$$

The half-space is conformally equivalent to a **strip** $\tau \in [0, \frac{1}{2}\beta]$ with boundary conditions $|B\rangle$. The path integral on the strip prepares a state $e^{-\beta\hat{H}/4}|B\rangle$ whose dual geometry is a **BTZ black hole with brane** at temperature β^{-1} [2, 3]. We continue $\tau \to t$ from $\tau = \frac{1}{4}\beta$.



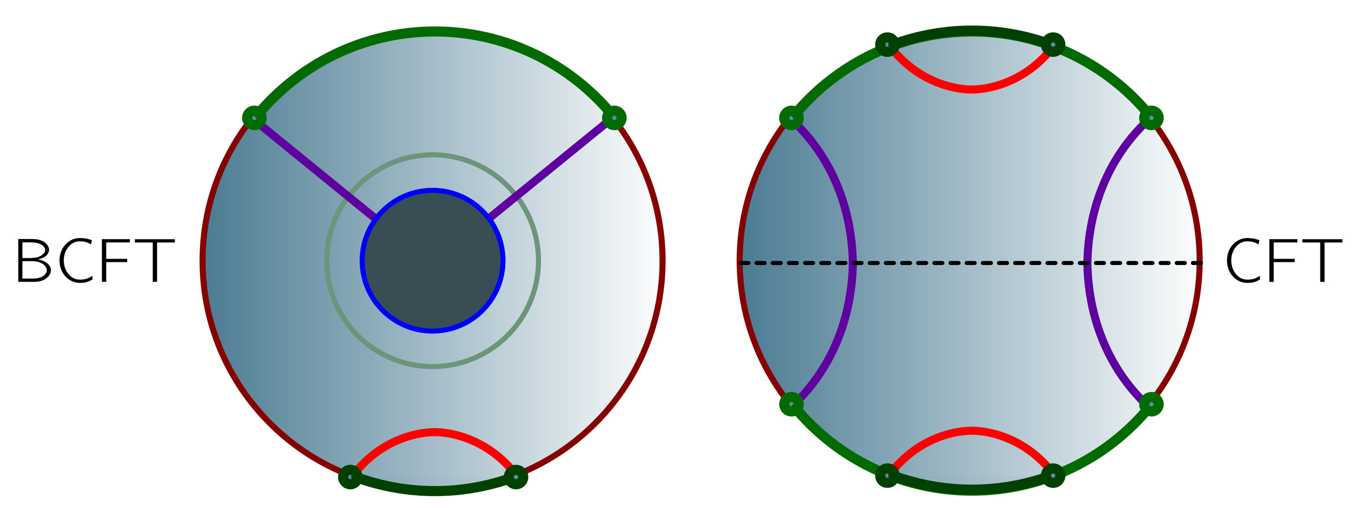


Holographic entanglement entropy

For a holographic state, the EE of an interval A is the length L of the extremal homologous **geodesic** in Planck units, $S_A = L/4G_N$ [4]. For the BTZ black hole, and $|A| = \ell$, we found

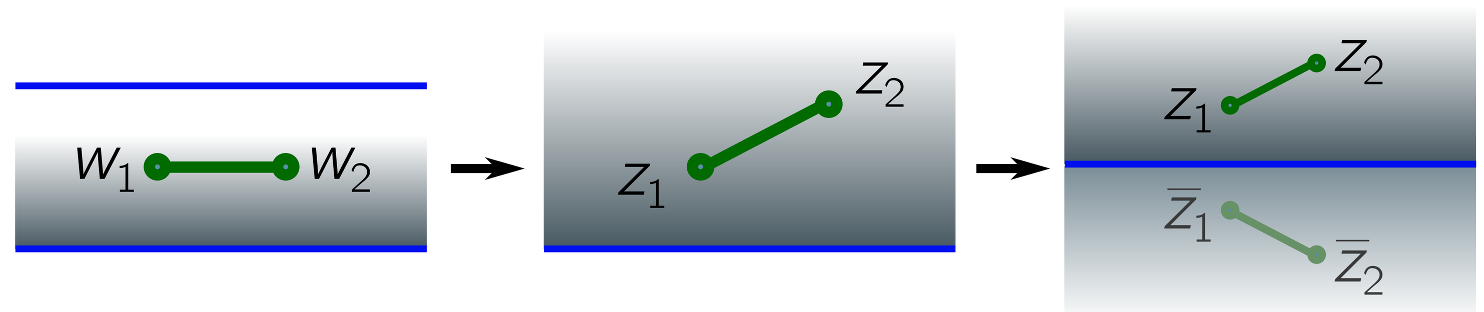
$$S_A(t) = \frac{c}{6}\min\left\{\ln\left[\frac{\beta}{\pi\epsilon}\sinh\left(\frac{\pi\ell}{\beta}\right)\right], \ln\left[\frac{\beta}{\pi\epsilon}\cosh\left(\frac{2\pi t}{\beta}\right)\sqrt{\frac{1+\ell_{\mathrm{AdS}}T}{1-\ell_{\mathrm{AdS}}T}}\right]\right\}.$$

Small intervals A give the thermal result. At large A, there is a **transition** to geodesics **ending on the brane**, and we can **probe behind the horizon** using EE. This resembles two intervals in CFT.



BCFT entanglement entropy

To calculate EE in the state $e^{-\beta\hat{H}/4}|B\rangle$, we first find Rényi entropies from a **correlator of twists** $\langle\Phi_n\bar{\Phi}_n\rangle$ and take $n \to 1^+$. Mapping the strip to the upper-half plane, and using the method of images, reduces our computation to a 4-point function on the plane [5].



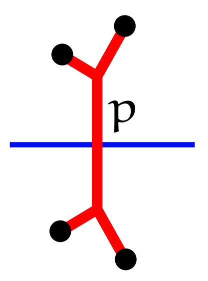
By symmetry, this correlator (for twist scaling dimension Δ_n) is

$$\langle\Phi_n(w_1)\bar{\Phi}_n(w_2)\rangle = \left(\frac{\pi}{2\beta}\right)^{2\Delta_n}\left[\frac{\eta\cdot z_{12}^2}{z_1 z_2}\right]^{-\Delta_n} F(\eta),$$

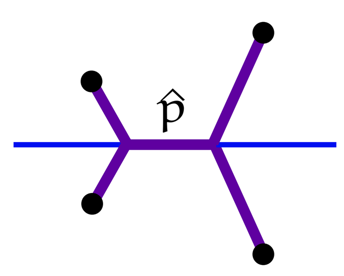
for some function $F(\eta)$ of the cross-ratio $\eta = z_{1\bar{1}}z_{2\bar{2}}/z_{1\bar{2}}$. The EE for the interval $A = [w_1, w_2]$ is $S_A = \lim_{n\to 1^+}(1-n)^{-1}\log\langle\Phi_n\bar{\Phi}_n\rangle$.

Matching entanglement entropies

The function $F(\eta)$ can be expanded in **conformal blocks** in the t-channel (fusion with OPE coefficients C) or s-channel (boundary operator expansion with coefficients B) [6, 7].



$$\sum_p C_{\Phi\bar{\Phi}p}\mathcal{F}^p(\eta)$$



$$\sum_{\hat{p}} B_{\Phi\hat{p}}B_{\bar{\Phi}\hat{p}}\mathcal{F}^{\hat{p}}(\eta)$$

Matching the holographic EE requires **vacuum block dominance** in both channels. To implement vacuum dominance, one might hope to adapt the large-c CFT conditions [7] to BCFTs:

- bulk and boundary blocks **exponentiate**, $\mathcal{F}^{p,\hat{p}} \approx e^{-(c/6)f^{p,\hat{p}}}$;
- the spectrum of bulk and boundary excitations is **gapped**.

However, understanding the behaviour of boundary blocks $\mathcal{F}^{\hat{p}}$ as $c \to \infty$ is the focus of ongoing work.

Future directions

There are several avenues for further investigation, including:

- extending the analysis to excited states;
- analysing the bulk replica geometry; and
- comparing to supersymmetric solutions.

References and acknowledgments

1. Affleck, Ludwig (1991).
2. Takayangi (2011); Miyaji, Takayanagi, Tonni (2011).
3. Almheiri, Mousatov, Shyani (2018); Almheiri (2018).
4. Ryu, Takayanagi (2006); Hubeny, Rangamani, Takayanagi (2007).
5. Cardy, Calabrese (2009); Cardy, Calabrese (2016).
6. Cardy, Lewellen (1991); Cardy (2004); Cardy, Tonni (2016).
7. Hartman (2013).

I would like to thank my colleagues, especially Mark Van Raamsdonk and Jamie Sully for close collaboration on the material presented here. I am indebted to Alex May, Dominik Neuenfeld, Chris Waddell, Tarek Anous and Eliot Hijano for discussions. I am supported by an International Doctoral Fellowship from the University of British Columbia.